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Heat transfer through a honeycomb sandwich panel

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HEAT TRANSFER THROUGH A HONEYCOMB SANDWICH PANEL

BY

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A

THESIS

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NOMENCLATURE

q - Heat transmitted through sandwich in Btu/Hr.

K - Thermal conductivity of metal in Btu/ Hr. Ft. ^{o}F.

A - Cross sectional area in Ft.^{2}

L - Length of the side of hexagon in Ft.

R - Radiation heat transfer from the sruface of hexagon in Btu/ Hr.

ϵ - Emissivity of metal.

E - Emissive power.

F - View factor.

B - Absorption factor.

σ - Stefan-Boltzmann's constant Btu/Hr. Ft.^{2} ^{o}R.

K_e - Effective thermal conductivity Btu/Hr. Ft. ^{o}F.

INTRODUCTION

Progress brought about by human ingenuity and technological developments has been always accompanied by fresh problems, the successful solution of which enables still further progress.

The recent developments in space exploration and exploitations are astounding. The coming years are likely to bring forth many remarkable achievments. Except for several engineering problems encountered with these developments the pace of progress is to study the heat transfer from honeycomb sandwich panels as an outer skin of high performance flight vehicles. The quantity of heat transferred through the panel is important in study of such vehicles, and methods or equations for determining this quantity are required.

Usually, the thermal resistance of the panel is experimentally determined by measuring the amount of heat transmitted through the panel in a steady-state condition with fixed face temperatures. The use of these experimental results however, is largely limited to the panels identical to those tested and in the same temperature range as the test temperatures. Since it is impractical to obtain extensive test data of all honeycomb core sandwiches of interest, it is desirable to have some method of analysis for this.

In the present investigation the equations are modified to permit the calculation of the steady-state temperature distribution duo to simultaneous conduction and radiation through honeycomb core sandwiches with given face temperatures. These equations can be solved for a range of face temperatures of interest.

REVIEW OF LITERATURE

Modern high performance flight vehicles employ honeycomb sandwich panels as an outer skin. Heat transfer through these panels has been the active consideration of engineers for only the past few decades. During this time little has been published on this subject.

Robert T. Swann (1)* calculated the effective thermal conductivities of square honeycomb sandwich panel for steady state heat transfer of the coupled modes of conduction and radiation which was published in Technical Note D - 171 of NASA, 1959. He concluded that the temperature distributions in the core were found to be essentially independent of the ratio of core height to cell size in the investigated range, and that the effective conductivity may be several times greater than the apparent conductivity of the sandwich, which is due to radiation.

M. C. Pittman and Robert T. Swann (2) calculated the effective thermal conductivities of honeycomb core and corrugated core sandwich panel. They determined some simple approximate equations, which are sufficiently accurate for most of the design calculations. They also developed a new procedure for all reflections of radiant energy. The effect of thermal radiation was found to increase approximately as a third power of the temperature, this of course is not comparable with the Stefan - Boltzmann's law. The effect of variable emissivity on effective thermal conductivity was investigated.

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* For all references refer to bibliography.

B. Gebhart (4) found an unified treatment for thermal radiation process for all diffuse radiation of gray surfaces. His paper was published in ASME papers 57A - 34, 1957. D. C. Hamilton and W. R. Morgan (3) investigated and plotted several new configuration factors including rectangle, triangle and cylinders of finite length which were published in Technical Note 2836 of NACA, 1952. Robert T. Swann (7) developed some general equations for the transfer of heat in sandwich panels in Technical Note D - 714 of NASA. In "Modern Developments in Heat Transfer", Warren Ibele mentioned three methods for the calculations of radiant energy between gray enclosure surfaces. These are (1) The Radiosity Method, (2) Hottel's Method and (3) Oppenheim's Electric Network Analog. F. Kreith (5) in "Radiation Heat Transfer for Spacecraft and Solar Power Plant Design" calculated the net radiation energy exchange of an enclosure with the help of determinants.

DISCUSSION

Cellular components of numerous varieties and materials are becoming popular as members of engineering structures on airplanes, boats, decks, etc. Their popularity is due to several of their unique properties such as extremely high strength to weight ratio, rigidity, low thermal conductivity, ease of manufacture, prefabrication of large components before final assembly, etc.

These cellular components usually take the form of cellular sandwich plates. These are really two plates with a cellular core securely bonded to and between them, similar to the one shown in Figure 1. The geometry of the cores or interior of the sandwich is varied. The one used here is not as numerous as some others.

The property in question in this thesis is the heat transfer characteristic of the structute and the relative importance of each of the modes, conduction, convection and radiation. It is of course possible to build a panel and test it, then change the structure and retest. This is time consuming and expensive. It is intended here to determine the expected performance analytically.

If effective thermal conductivity,,which includes the effect of all modes of heat transfer, can be determined for honeycomb sandwith panels whose faces are at different but constant temperatures, and the amount of heat transmitted through the panel can be calculated. Since air spaces normally exist in such panels, it is necessary to consider radiation between surfaces and convection and conduction through air as well as conduction through the cells.

The heat transferred by a combination of modes is a nonlinear

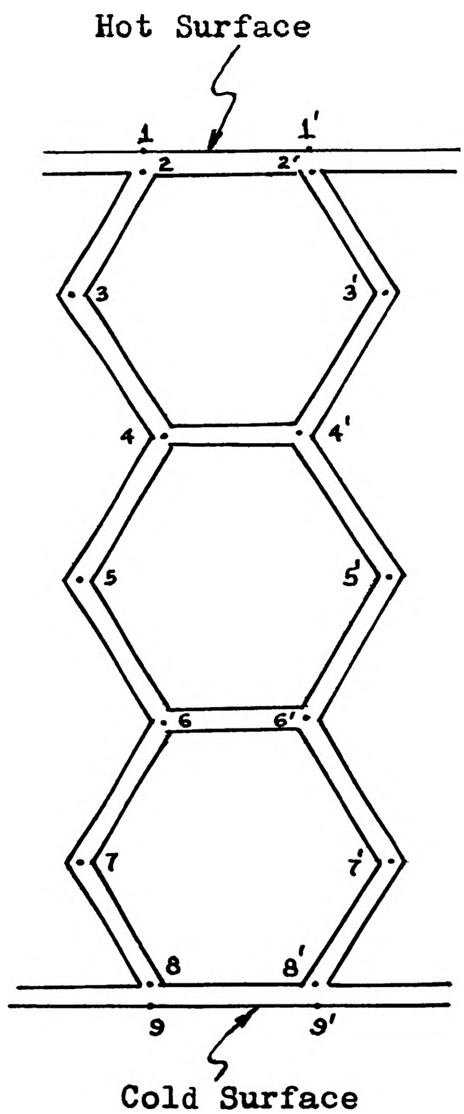


Figure. 1 Section of a Honeycomb Sandwich showing the locations of parts.

function of temperature. All modes of heat transfer are interrelated. The amount of heat transferred by convection however, is usually small when compared to that transferred by conduction and radiation and can be considered independent of these.

In this analysis heat is added to one face and removed at the other face of the sandwich. With the help of the face temperatures, a sufficient number of equations is available to determine the temperature distribution across the honeycomb sandwich panel.

Radiation analysis is difficult across the honeycomb sandwich panel because no view factor is known. In this analysis end effects of radiation at the ends are neglected because the passages are sufficiently long.

Let the given honeycomb sandwich be divided into m parts as shown in figure 1 . Since conduction in this case takes place in one direction only as shown in the figure 2 , it must follow Fourier's conduction equation:

$$Q = \frac{- K A (t_2 - t_1)}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

where Q is the amount of heat flow,

A is the area normal to the direction of heat flow,

K is the thermal conductivity of the material,

t_2 is the temperature of the hot side,

t_1 is the temperature of the cold side,

and L is the thickness of the material.

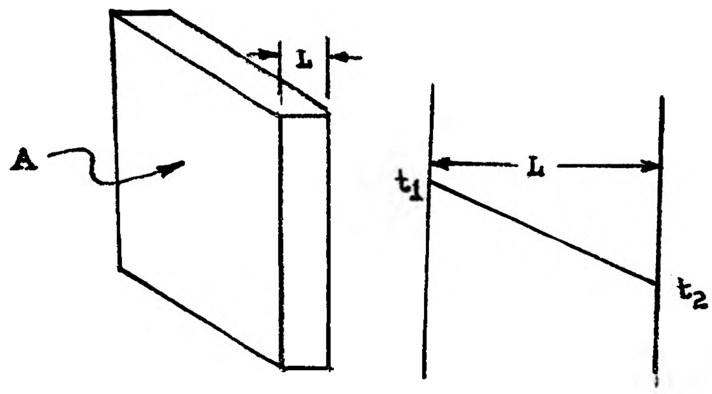


Fig. 2

One-dimensional Steady-state heat conduction

Temperature decreased with position in the direction of heat flow; therefore, if heat flow is taken as positive, the temperature gradient must be negative as shown in equation (1).

For the purpose of analysis it is necessary to make some ground rules better known as assumptions. These in reality are for the purpose of defining the problem, excluding some possiblities which the author believes are of minor importance, and in most cases making the problem simple enough to be one that the author can manage.

The assumptions here are as follows:

(a) Uniform thermal conductivity of the metal of the structure throughout the entire temperature range according to Fourier's law;

(b) The faces of the cells, for the purpose of computing the radiation only, are assumed at uniform temperature across the interspace;

(c) One directional steady heat flow by conduction.

The temperature distribution across the honeycomb, figure 1, is given below:

$$q_{1-2} - \frac{K A_{1-2} (t_1 - t_2)}{L_{1-2}} = 0$$

$$q_{2-3} - \frac{K A_{2-3} (t_2 - t_3)}{L_{2-3}} + R_{2-3} = 0$$

$$q_{3-4} - \frac{K A_{3-4} (t_3 - t_4)}{L_{3-4}} + R_{3-4} = 0$$

$$q_{4-5} - \frac{K A_{4-5} (t_4 - t_5)}{L_{4-5}} + R_{4-5} = 0$$

$$q_{5-6} - \frac{K A_{5-6} (t_5 - t_6)}{L_{5-6}} + R_{5-6} = 0$$

$$q_{6-7} - \frac{K A_{6-7} (t_6 - t_7)}{L_{6-7}} + R_{6-7} = 0$$

$$q_{7-8} - \frac{K A_{7-8} (t_7 - t_8)}{L_{7-8}} + R_{7-8} = 0$$

$$q_{8-9} - \frac{K A_{8-9} (t_8 - t_9)}{L_{8-9}} = 0$$

where q is the amount of heat flow,

K is the thermal conductivity of the metal,

L is the length of one side of the hexagon,

t is the temperature in $^{o}R.$,

and R is the radiant heat exchange between the inner faces of the hexagons.

The subscripts indicate the points between which the heat flows as from 1-2, ... 7-8, etc.

So total heat transfer from face 1 to 9 will be:

$$q_{total} = \frac{K A_{1-9} (t_1 - t_9)}{L_{1-9}} - R_{total} \qquad \text{..........} \ 3$$

It is desirable to determine the net radiation to (or from) surfaces of honeycomb sandwich panel. By symmetry heat transfer from face 1' to 9' will be same as from 1 to 9, so if the radiation from inner faces of hexagon can be found, the total radiation from the honeycomb sandwich panel can be calculated.

While considering the radiant energy, it is necessary to discuss some fundamentals of it. Thermal radiation is energy emitted by matter in the form of electromagnetic waves. The energy emission arises because of changes in thermal energy states of the microscopic particles of which the material is composed.

The net rate of heat transfer among a group of radiating subjects depends upon the intervening medium, upon each material and its temperature, and upon the geometric relation of the objects to one another. However, the the rate of emission from any object depends only upon the material and its state.

If surface is "black", that is, if it absorbes all incident radiation and therefore appears black, the emission rate per unit area is given by the Stefan - Boltzmann law

$$e_b = \sigma T^4$$

where e_b is emissive power,

σ is Stefan - Boltzmann constant which is equal to 0.1713×10^{-8} Btu/ Ft.^{2} Hr. $^{\circ}$R.^{4}

and T is absolute temperature in $^{\circ}$F.

Most actual surfaces emit at a lower rate than does a black surface, at the same temperature. The hemispherical emissive power is therefore

$$e = \epsilon e_b = \epsilon \sigma T^4$$

where ϵ is the hemispherical emissivity of the material.

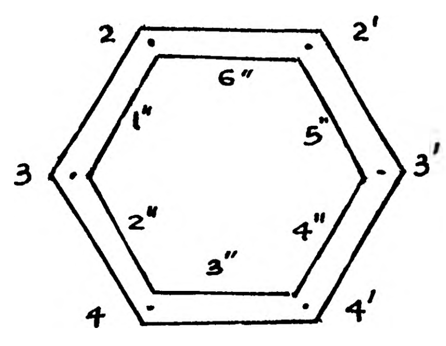


Fig. 3

Hexagon of Honeycomb sandwich showing the position of points.

For diffuse-radiation, B. Gebhart has presented a method, To write the equation for the rate of net radiant energy transfer for a surface, its entire surrounding must be specified. The rate of net heat transfer from a typical surface, j, to all other surfaces in the enclosure requires an energy summation (or balance) on the j th surface. Considering an enclosure, which contains no enclosure emitting or absorbing media, formed of areas $A_{1''}$, $A_{2''}$ . .... having emissivities $\epsilon_{1''}, \epsilon_{2''}, \ldots$ and emissive power $E_{1''}$, $E_{2''}$ .. ... ., where 1", 2", 3" .... indicate inner surfaces of a hexagon respectively as shown in figure 3 . The net rate of radiant heat loss q_j is equal to the rate of heat emitted by A_j minus the total rate of radiant energy absorbed by A_j from all sources. All the surfaces of the enclosure may emit radiant energy. The amount of heat absorbed by A_j, after direct radiation or reflection or multiple reflection, must be summed. The i^{th} surface **emits** heat at the rate of $E_i A_i$, and **a portion of this** energy may arrive at A_j by many complicated multiple reflection processes. A net absorption **fa**ctor B_{ij} is defined as the total fraction of the radiant energy emitted by A_1 and absorbed by A_j..

The net rate of loss of thermal energy by radiation from A_j surface is

$$q_j = \epsilon_j \sigma T_j^4 A_j - (\epsilon_{1''} \sigma T_{1''}^4 A_{1''}) B_{1''j} - \ldots\ldots$$

$$\ldots\ldots - (\epsilon_{6''} \sigma T_{6''}^4 A_{6''}) B_{6''j} .$$

$$= E_j A_j - E_{1''} A_{1''} B_{1''j} - \ldots\ldots - E_{6''} A_{6''} B_{6''j}$$

$$= E_j A_j - \sum_{i=1''}^{6''} B_{ij} E_i A_i$$

where EA denotes the rate at which energy is radiated from the surface (other than reflection) and is merely the hemispherical emissive power $\epsilon A T^4$, where ϵ is the total hemispherical emissivity.

The following is the method for determining the absorption factors; B_{ij}. Considering the paths through which energy is emitted by surface 1" may impinge on 2" and from which a part may be reflected from 2" to surface j. The amount of such energy reflected from 2" is

$$\rho_{2''} F_{1''-2''} (\epsilon_{1''} \sigma T_{1''}^4 A_{1''}),$$

Under the condition that energy emitted or reflected from 2" has the same directional distribution, it follows that:

$$B_{2''j} \rho_{2''} F_{1''\to 2''} (\epsilon_{1''} \sigma T_{1''}^4 A_{1''})$$

is absorbed at surface j. Similarly the energy from 1" may strike 3" and thence is absorbed by j and will amount to:

$$B_{3''j} \rho_{3''} F_{1''-3''} (\epsilon_{1''} \sigma T_{1''}^4 A_{1''})$$

for all surfaces in the enclosure. It may be noted that if surface 1" is concave, a fraction $F_{1''\to 1''}$ of the emergy emitted by 1 strikes itself, and of that:

$$B_{1''j} \rho_{1''} F_{1''\to 1''} (\epsilon_{1''} \sigma T_{1''}^4 A_{1''})$$

is absorbed by j. The summation gives the energy absorbed at j due to the emission of 1". In general, the total fraction of $A_{1''}E_{1''}$ absorbed at A_j, that is $B_{1''j}$, is:

$$B_{1''j} = F_{1''j} \epsilon_j + F_{1''-1''} \rho_{1''} B_{1''j} + \ldots\ldots + F_{1''6''} \rho_{6''} B_{6''j}$$

similarly, for each of the rest of the surfaces,

$$B_{2''j} = F_{2''-j}\,\epsilon_j + F_{2''-1''}\,\rho_{1''}\,B_{1''j} + \cdots\cdots F_{2''-6''}\,\rho_{6''}\,B_{6''j}$$

$$\vdots$$

$$B_{6''j} = F_{6''-j}\,\epsilon_j + F_{6''-1''}\,\rho_{1''}\,B_{1''j} + \cdots\cdots\cdots F_{6''-6''}\,\rho_{6''}\,B_{6''j}$$

which will be

$$B_{kj} = F_{k-j}\,\epsilon_j + \sum_{i=1''}^{6''} \rho_i\, F_{k-i}\, B_{ij} \qquad \cdots\cdots\cdots \quad 6$$

where $K = 1'', 2'', 3'' \cdots\cdots\cdots 6''$.

This set of 6 equations may be solved for 6 unknowns $B_{1''j}$, $B_{2''j}$, ... $B_{6''j}$. The above set of equations is valid, with the same coefficients, for any choice of j, that is 1", 2", 3" .... 6". Therefore equation (6) can be applied to any surface of enclosure.

There are certain necessary relations between those 6 x 6 absorption factors of six surface enclosure, because there are six by six view factors.

Since the whole emission of A_i is absorbed by $A_{1''}$, $A_{2''}$ ... . $A_{6''}$

$$\sum_{j=1''}^{6''} B_{ij} = 1 \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad 7$$

where $i = 1'', 2'', 3'', \;.. \;6''$.

Furthermore, at thermal equillibrium and by reciprocity

$$B_{ij}\,\epsilon_i\,A_i = B_{ji}\,\epsilon_j\,A_j \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad 8$$

The expression for the net rate of radiant energy loss from any surface j of the hexagon may be written by

$$q_j = \sigma \epsilon_j A_j \left(T_j^4 - \sum_{i=1''}^{6''} B_{ji} T_i^4 \right) \quad \cdots\cdots\cdots \quad 9$$

If the effective thermal conductivity for sandwich panel is K_e then heat transfer through honeycomb sandwich would be

$$q_n = \frac{K_c \ (t_1 - t_2)}{L} A \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad 10$$

where q_n is net heat transfer of combined modes of conduction and radiation which can be calculated with the help of equations (2) and (9),

and L is the thickness of the hoenycomb sandwich.

Then:

$$K_e = \frac{q_n \ L}{(t_1 - t_9) \ A} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad 11$$

It would be well to illustrate the use of this with a practical problem.

Consider an aluminum honeycomb sandwich panel which has

Emissivity = = 0.2

Conductivity of Aluminum = 144 Btu/Hr. Ft. ^{o}F.

Temperature t_1 = 1000^{o}R.

Temperature t_9 = 500^{o}R.

= 550^{o}R.

= 600^{o}R.

Lengths of one face of the hexagon

= 0.4"

= 0.5"

= 0.6"

Thickness of the plate = 0.0625"

Temperature distribution across honeycomb sandwich of 0.5 inch side hexagon will be as shown in table I.

Distance from upper surface in inch	Temperature ^{o}R.
0.0000	1000
0.0625	998.3
0.437	934
0.87	846.5
1.305	758.5
1.74	673.5
2.18	589
2.587	506
2.61	504.3

TABLE I

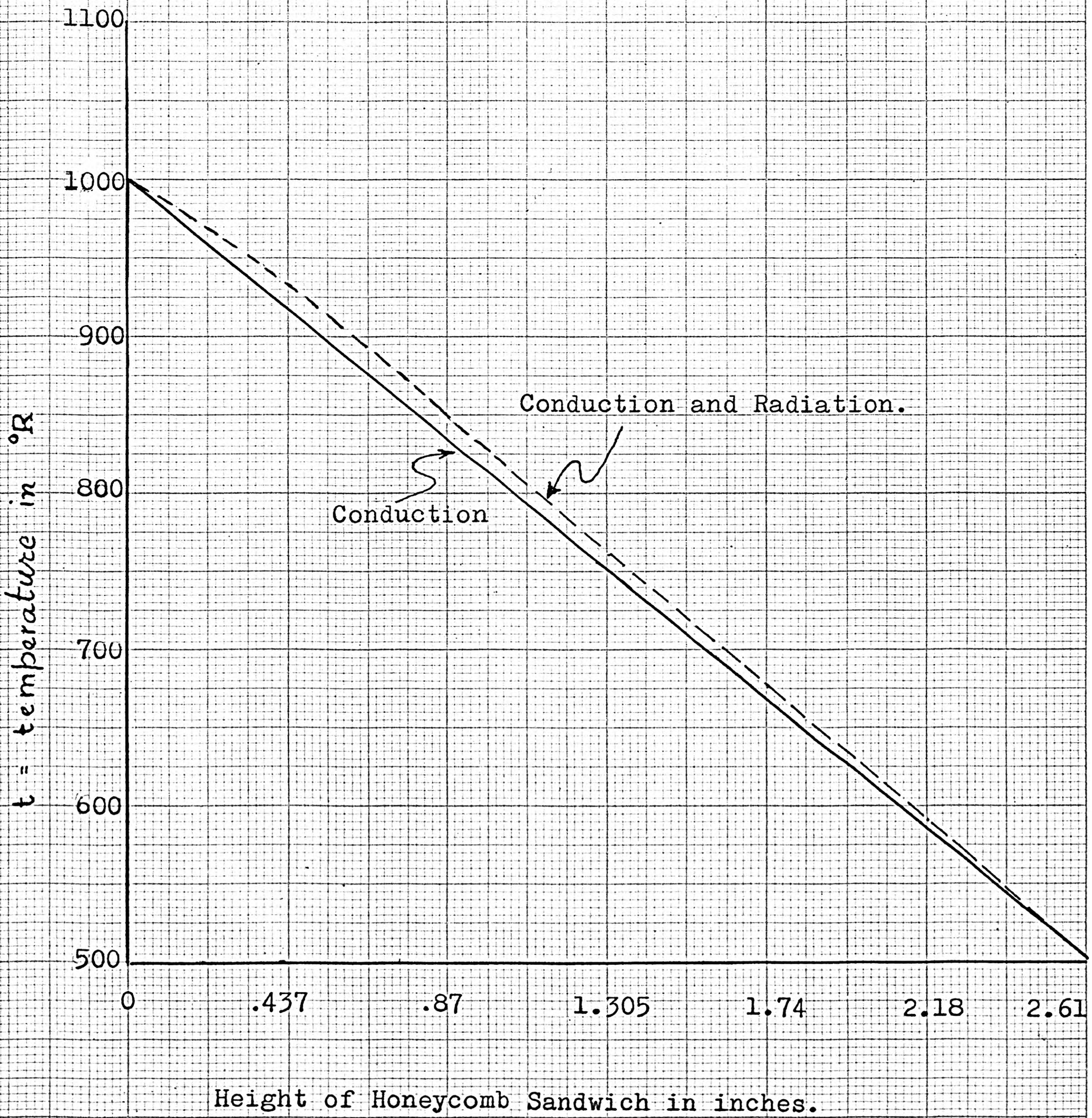
Effective thermal conductivity will be as shown in table II

T Hot °R.	T Cold °R.	K_e for honeycomb sandwith of 0.4" side hexagon	0.5" side hexagon	0.6" side hexagon
1000	500	20.02	16.32	12.80
	650	20.06	16.34	12.83
	600	20.09	16.38	12.87
	700	20.12	16.42	12.97

TABLE II

Therefore q_{net} for 0.5" side hexagon will be 670 Btu/Hr. Ft. °F.

Figure. 4 Steady-state temperature distributions through honeycomb sandwich.



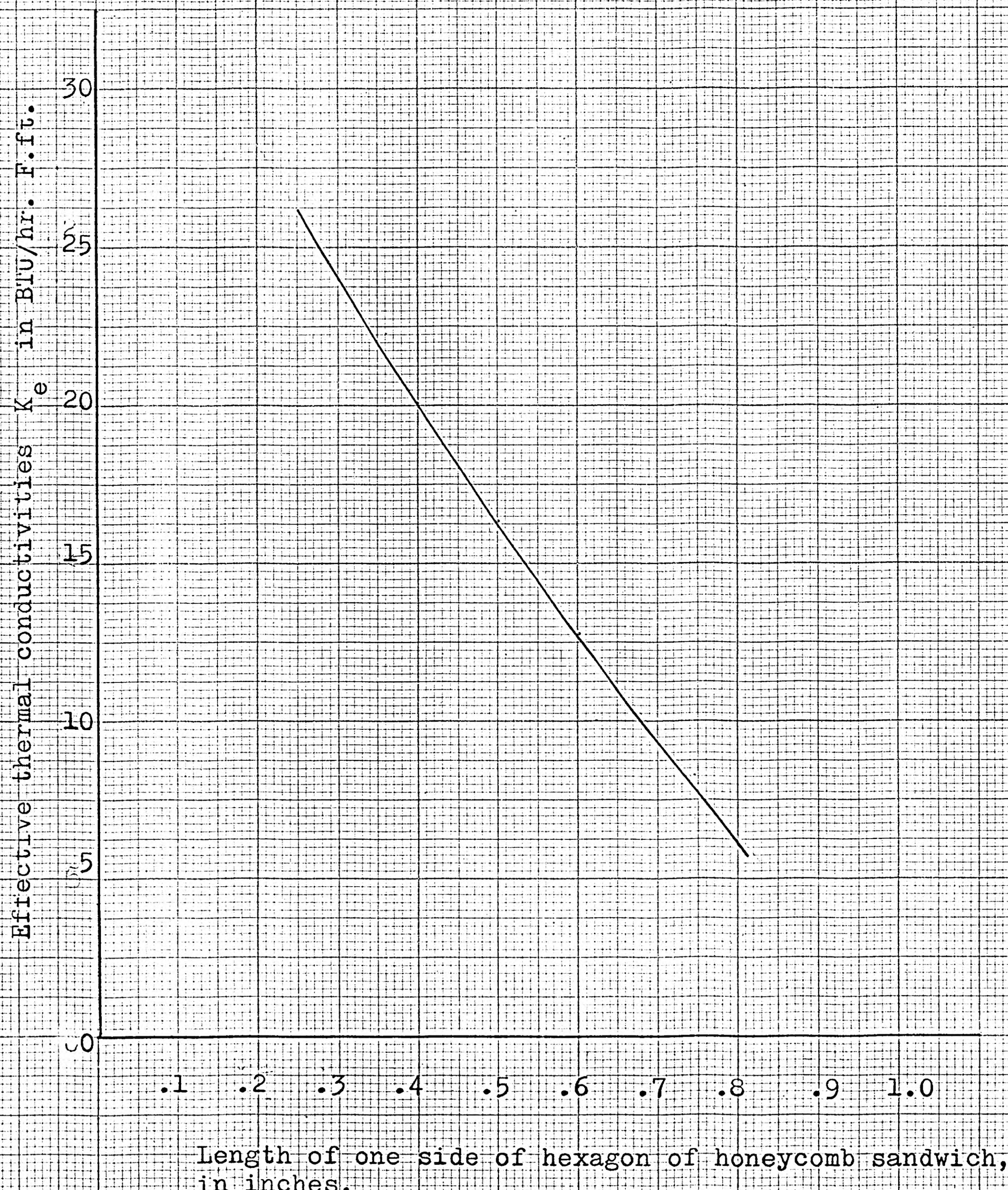


Figure 5. Effective conductivities of honeycomb sandwich.

CONCLUSION

The analytical solution has been made to determine the effective thermal conductivity of honeycomb sandwich panels in which steady state heat transfer takes place by the coupled modes of conduction and radiation. The results of the investigation could be reduced to following conclusions:

(1) The effect of combined radiation and conduction is to separate the temperature distribution from the linear distribution which results from simple conduction theory.

(2) The interaction leads to increase the temperature gradient at the cold face and correspondingly heat transfer by conduction.

(3) The effective conductivity may be more than apparent conductivity of the sandwich in which heat transfer by radiation is assumed absent.

(4) The effective thermal conductivity is less than the metal conductivity due to air gaps.

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VITA

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APPENDIX

VIEW FACTORS

The analysis of above discussion requires view factors for the calculation of the radiant energy interchange. These factors are alternatively known as angle factors, geometrical factors, shape factors, configuration factors or view factors, and represent the fraction of the energy, leaving one surface which is incident upon other surface. The mathematical relation can be obtained by figure 6.

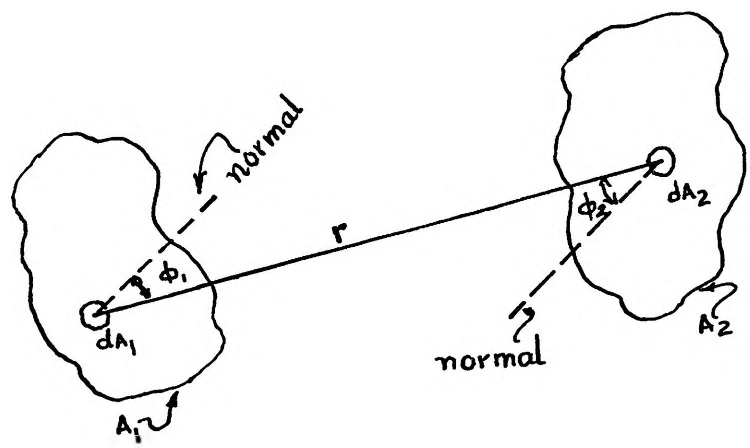

Fig. 6.

Determination of **View Factor**

The radiation energy leaving dA_1 and impinging on dA_2 is:

$$dq_{dA_1-dA_2} = I\, d\omega\, dA_1$$

where I is the intensity of the radiant energy,

$d\omega$ is the solid angle subtended by surface 2 from the

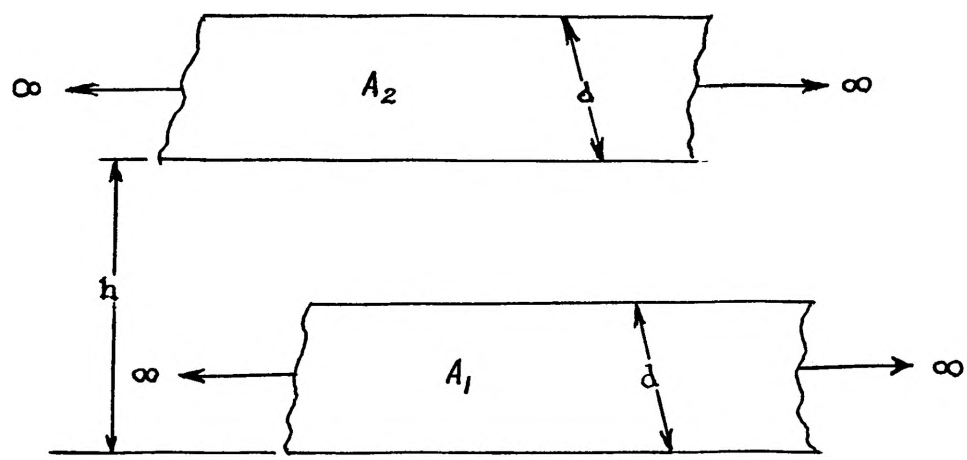

Figure. 7 Two infinitely long parallel planes.

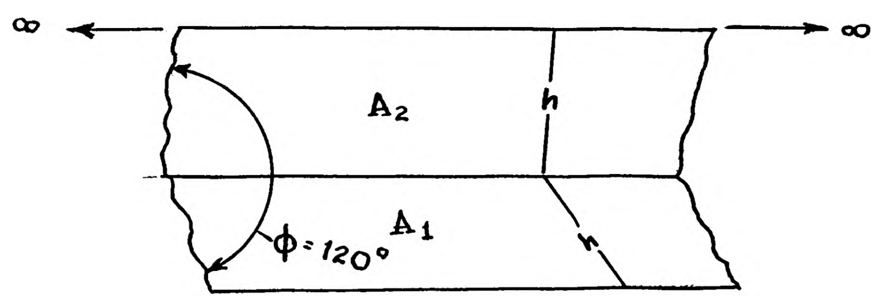

Figure. 8 Two infinitely long plane intersecting at an angle $\phi = 120°$

For diffuse radiation

$$I_n = \epsilon \frac{E_b}{\pi}$$

$$dF_{dA_1 - dA_2} = \frac{dq_{dA_1 - dA_2}}{E_1 \; dA_1} = \frac{\cos\phi_1 \; \cos\phi_2 \; dA_2}{\pi r^2}$$

Then, $E_{1-2} = e_{b1} \frac{1}{\pi} \int_{A_1} \int_{A_2} \frac{\cos\phi_1 \; \cos\phi_2 \; dA_2 \; dA_1}{r^2}$

For convenience, a view factor F_{1-2} may be defined by equation

$$A_1 \; F_{1-2} \equiv \frac{1}{\pi} \int_{A_1} \int_{A_2} \frac{\cos\phi_1 \; \cos\phi_2 \; dA_2 \; dA_1}{r^2}$$

D. C. Hamilton and W. R. Morgan gave view factors for infinitely long parallel identical, directly opposed rectangles, and for two rectangle plates with one common edge as below :

$F_{1-2} = \frac{1}{2} \left[\sqrt{1 + \left(\frac{h}{d}\right)^2} - \frac{h}{d} \right]$ for two infinitely parallel planes and

$F_{1-2} = \frac{1}{2} (1 + \cos\phi)$ for two infinitely long planes intersecting at an angle ϕ ; as:

$$\sum_{i=1''}^{6''} F_{ij} = 1$$

Thus all the view factors could be computed.